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SYMMETRIC ROUND TRIP FLYBYS TO OUTER PLANETS

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An exploratory analysis was performed to determine the feasibility of outer planet, round-trip mission profiles which might have manned flight applications. Simplifying assumptions used minimized the analytical effort, but the main conclusions are still considered valid. A round-trip flyby of Jupiter of approximately 1000 days duration was found, which appears to be within the capabilities of technology generally assumed for a manned Mars stopover mission. Missions beyond Jupiter were found to be either of very long duration or very demanding in terms of propulsion requirements. Multiplanet flybys were not examined, but it is suggested that, at least in the case of a Jupiter-Saturn-and-return flyby, energy requirements might be less than for the Saturn-only case.

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

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ADVANCED SYSTEMS OFFICE
RESEARCH AND DEVELOPMENT OPERATIONS

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DEFINITION OF SYMBOLS

Symbols are listed in subscripted form (rather than a separate list of subscripts) in order to assure indication of correct meaning.

Symbol	Meaning
$\mathcal{E}$	Eccentric anomaly
EMOS	Earth's mean orbital speed (29.8 km/sec)
n	Integral number of Earth years during mission (e.g., for a 5.67-year mission n = 5)
$P_{\oplus}$	Duration of one Earth year
R	Radius (in km)
R_{ar}	Radius from target planet center at which apsidal rotation maneuver occurs
R_p	Perihelion radius from sun (1 A. U., 149.6×10^6 km)
R_{pmin}	Minimum allowed periapsis radius at target planet
R_{po}	Radius of parking orbit around Earth
R_{pt}	Periapsis radius at target planet
R_t	Radius of target planet's orbit from sun
V	Velocity in km/sec
V_{ar}	Planetocentric velocity at which apsidal rotation maneuver occurs
$V_{a\oplus}$	Aerodynamic entry velocity at Earth return
V_c	Planetocentric velocity for "just captured" at target planet (parabolic speed)

DEFINITION OF SYMBOLS (Continued)

Symbol	Meaning
V_{cc}	Planetocentric velocity for circular capture at R_{pt} at target planet
V_{HP}	Planetocentric approach velocity
$V_{HP\oplus}$	Hyperbolic excess velocity at Earth departure.
V_P	Heliocentric perihelion velocity for transfer
V_{PH}	Perihelion velocity for Hohmann transfer to target
V_{PO}	Velocity in Earth parking orbit
V_{PT}	Planetocentric periapsis velocity at target planet
V_T	Target planet mean orbital speed
V_x, V_y, V_z	Components of velocity used in transferring from heliocentric to planetocentric coordinates
$V_{\oplus}'$	Velocity at Earth parking orbit radius to produce $V_{HP\oplus}$
$V_{\theta T}$	Heliocentric approach velocity at target
ΔV	Propulsive velocity change (assumed impulsive)
ΔV_{ar}	ΔV required for apsidal rotation
ΔV_c	ΔV for marginal capture at target planet
ΔV_{cc}	ΔV for circular capture at target planet
$\Delta V_{\oplus}$	ΔV for Earth departure
β	Half of angle between asymptotes of approach hyperbola at target

DEFINITION OF SYMBOLS (Concluded)

Symbol	Meaning
ϵ	Orbit eccentricity for transfer
ϵ_H	Eccentricity for Hohmann transfer from Earth to target planet
ϵ_t	Eccentricity of approach hyperbola at target planet
θ	Heliocentric angle of transfer from Earth to target planet
ξ	Planetocentric approach angle defined by equations (19) & (20)
$\mu_{\odot}$	Solar gravitational constant, km^3/sec^2
$\mu_{\oplus}$	Earth's gravitational constant
μ_t	Target planet gravitational constant
τ	Duration of transfer
τ_H	Duration of Hohmann transfer
ϕ	Heliocentric approach angle
ϕ'	Planetocentric approach angle
ϕ_{ar}	Planetocentric path angle at point of apsidal rotation maneuver
ψ_r	Required angle of apsidal rotation

SYMMETRIC ROUND TRIP FLYBYS TO OUTER PLANET

SUMMARY

An exploratory analysis was performed to determine the feasibility of outer planet, round-trip mission profiles which might have manned flight applications. Simplifying assumptions used minimized the analytical effort, but the main conclusions are still considered valid. A round-trip flyby of Jupiter of approximately 1000 days duration was found which appears to be within the capabilities of technology generally assumed for a manned Mars stopover mission. Missions beyond Jupiter were found to be either of very long duration or very demanding in terms of propulsion requirements. Multiplanet flybys were not examined, but it is suggested that, at least in the case of a Jupiter-Saturn-and-return flyby, energy requirements might be less than for the Saturn-only case.

PURPOSE

This report communicates results of an exploratory investigation into outer planet missions. The purpose of the investigation was to arrive at initial estimates of the level of technology that might be required for manned missions to the outer planets and to select certain mission profiles for more precise analysis.

BACKGROUND

The powerful gravitational field of Jupiter has long been recognized as a potentially effective means of trajectory shaping for space missions. Stewart [1] has discussed a variety of possible space probe missions based on Jupiter flybys, and others have been studied at MSFC [2]. The specific concept of using Jupiter flyby to facilitate Earth return is not mentioned in either of these references, although the analytical methods used are applicable [3]. For an unmanned mission there is probably no motivation to attempt an Earth-return mission. However, a "free" (i.e. nonpropulsive) Earth return might make a Jupiter flyby a feasible manned mission if the duration were not too great.

The analysis herein reported was undertaken to assess the flight mechanics feasibility of a manned Jupiter flyby. Simplifying assumptions, as noted in the following section, were used to facilitate a straightforward analysis. For completeness, mission profiles to the other outer planets were determined, although attractive manned mission possibilities were not expected.

ANALYSIS

The concept of the symmetric outer planet flyby shown in Figure 1 illustrates the case for Jupiter. The gravitational field of the planet deflects

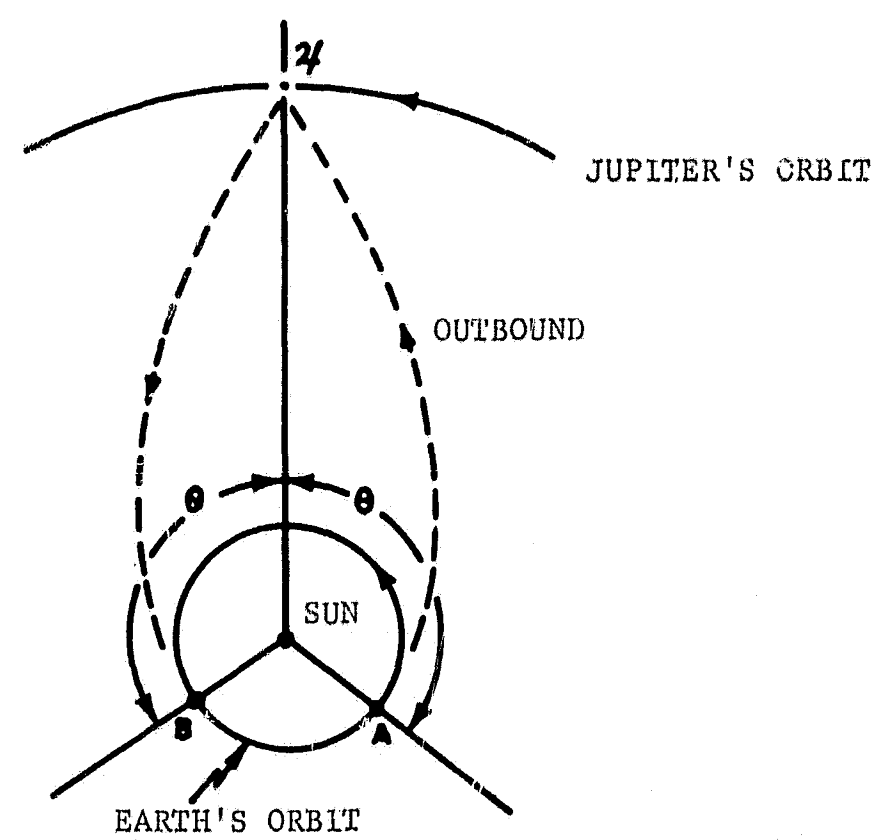


FIGURE 1. SYMMETRIC JUPITER FLYBY CONCEPT

the trajectory of the spacecraft so that it returns to Earth. If the gravitational deflection is insufficient, propulsion may be employed to assist and effect Earth return. The following simplifying assumptions minimized the analytical effort required:

1. Planetary orbits are circular and coplanar.

2. Departures from and arrivals at Earth are tangential to Earth's orbit.

3. The analytical method consisted of "patched conics" (which assumes that planetary and solar gravitational fields may be considered independently).

4. Outbound paths are symmetrical with inbound paths.

These trajectories are classified type I; therefore, we may be sure that all possible transfers will be of shorter duration than the Hohmann transfer. The minimum duration trip is one for which the Earth simply travels from point A to point B during the mission (less than one year). Other missions will be found where the Earth makes one or more complete revolutions during the mission. A round trip mission requires that Earth be at point A on departure from Earth and at point B on return to Earth. Approximate hand calculations made for the Jupiter case illustrate the nature of the solutions (Fig. 2), which occur where the "spacecraft" and "Earth" curves intersect.

The following analysis gives the equations and sequence of solution without derivations. The first step is to establish the duration of the Hohmann transfer which will set an upper limit to the durations of possible missions of this type:

$$\epsilon_H = \frac{2R_t}{R_t + R_p} - 1 \tag{1}$$

$$\tau_H = \frac{\pi \left(\dfrac{R_p}{1 - \epsilon_H}\right)^{3/2}}{\sqrt{\mu_\odot}} . \tag{2}$$

As the flyby missions go to longer durations, approaching Hohmann conditions, the angle θ approaches π, and if n is the integral number of complete revolutions made by the Earth during the missions, the total duration approaches:

$$2\tau \approx (n + 1) P_\oplus \tag{3}$$

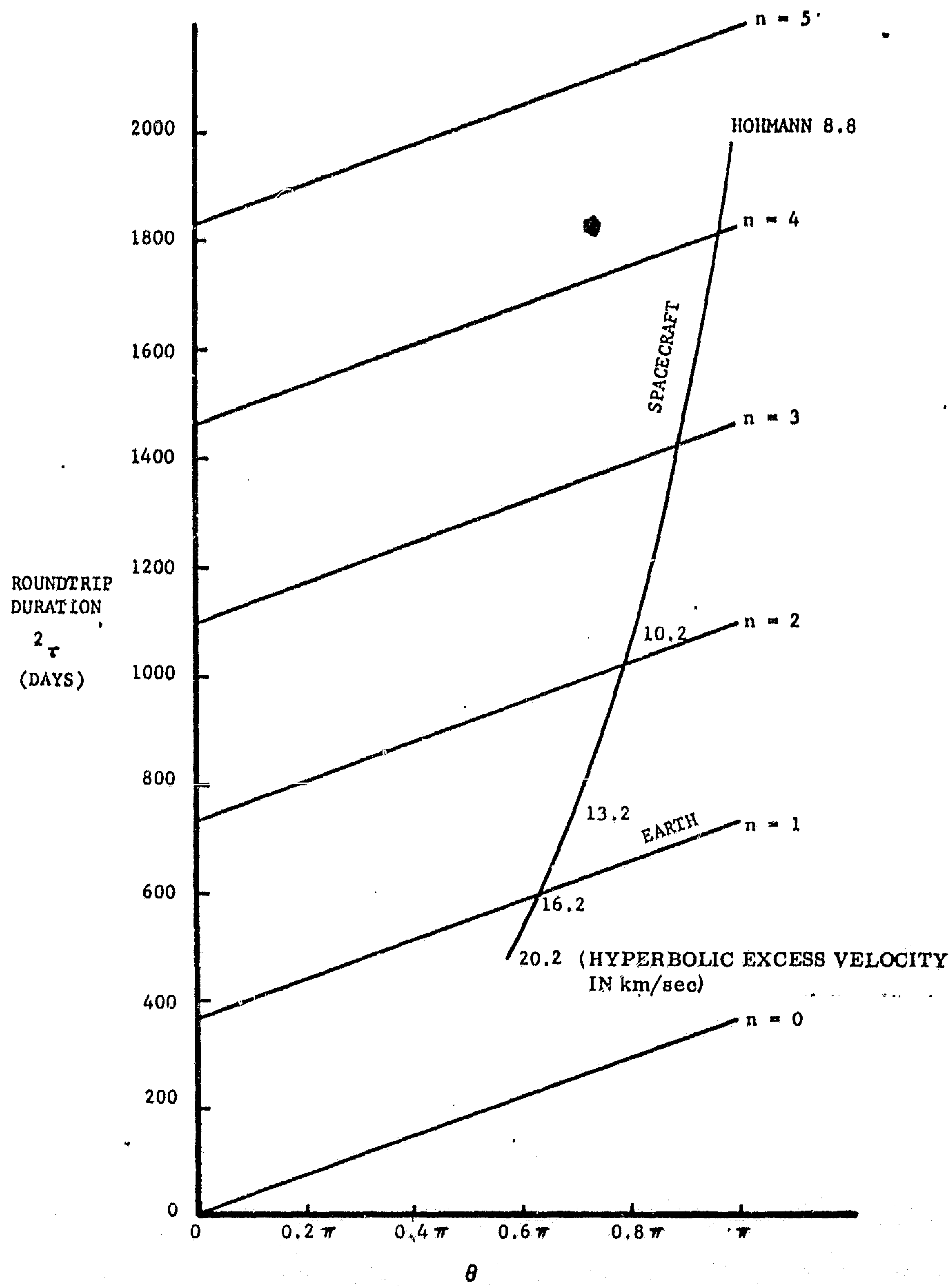


FIGURE 2. JUPITER FLYBY SOLUTIONS

Therefore, if we select a value, n, such that

$$(n + 1)P_\oplus > 2\tau_H > nP_\oplus$$

and we desire that $2(n_{max}\pi + \theta)P_\oplus < 2\tau_H$ (thus setting the longest mission at a shorter duration than $2\tau_H$), and noting $\theta \approx \pi$, we find it to be very likely true that $n_{max} = n - 2$.

Further, it may be noted that, since the Hohmann mission is minimum energy, the perihelion velocities for all the flyby missions will be higher than for the Hohmann mission:

$$V_{PH} = \sqrt{\frac{\mu_\odot(\epsilon_H + 1)}{R_p}} \quad . \tag{4}$$

$$\text{Then, always, } V_P > V_{PH} \quad . \tag{5}$$

An iterative solution is required to find V_P for any value of n. It is convenient to begin with $V_P = V_{PH} + 1$ where velocities are in km/sec.

Eccentricity of the transfer is computed by:

$$\epsilon = \frac{R_p V_P^2}{\mu_\odot} - 1 \quad . \tag{6}$$

Heliocentric angle traversed during transfer to the target planet's orbit is given by:

$$\cos\theta = \frac{R_p^2 V_P^2/\mu_\odot R_t - 1}{\epsilon} \quad . \tag{7}$$

The computer performs an inverse tangent routine to derive inverse functions. Therefore, we set:

$$\tan\theta = \frac{\sqrt{1 - \cos^2\theta}}{\cos\theta} \quad . \tag{8}$$

As tan θ ranges from $-\infty$ to ∞, the value of θ determined by the computer will range from $-\pi/2$ to $\pi/2$. Therefore, a test routine is used such that if cos θ is negative, values of θ between $\pi/2$ and π are generated.

Another test is used prior to calculating transfer time to determine whether the transfer is elliptic or hyperbolic. If $\epsilon < 1$, the transfer is elliptic, and we find a parameter, $\mathcal{E}$, from

$$\cos \mathcal{E} = \frac{1}{\epsilon} - \frac{R_t}{R_p}\left(\frac{1-\epsilon}{\epsilon}\right) \quad . \tag{9}$$

$\mathcal{E}$ is found from cos $\mathcal{E}$ by the inverse tangent routine. Transfer time is then found from

$$\tau = \frac{\left(\frac{R_p}{1-\epsilon}\right)^{3/2}}{\sqrt{\mu_\oplus}} \left(\mathcal{E} - \epsilon \sin \mathcal{E}\right) \quad . \tag{10}$$

If $\epsilon > 1$ the transfer is hyperbolic and $\mathcal{E}$ is found from

$$\cosh \mathcal{E} = \frac{1}{\epsilon} + \frac{R_t}{R_p}\left(\frac{\epsilon-1}{\epsilon}\right)$$

and

$$\tau = \frac{\left(\frac{R_p}{\epsilon-1}\right)^{3/2}}{\sqrt{\mu_\oplus}} \left(\epsilon \sinh \mathcal{E} - \mathcal{E}\right) \quad . \tag{11}$$

We then set $\tau = P_\oplus(n + \frac{\theta}{\pi})$ and iterate on V_P to get $\tau = \tau_s$. The iteration scheme attempts to use a Newton-Raphson method; if this fails by setting $V_P < V_{PH}$, as is rather likely for near-Hohmann conditions, a decreasing-increment search is used. The iteration is converged when

$$|\tau - \tau_s| \leq 100\,000 \text{ sec} \qquad \text{(slightly more than 1 day)}.$$

The remainder of the calculations may now be done directly.

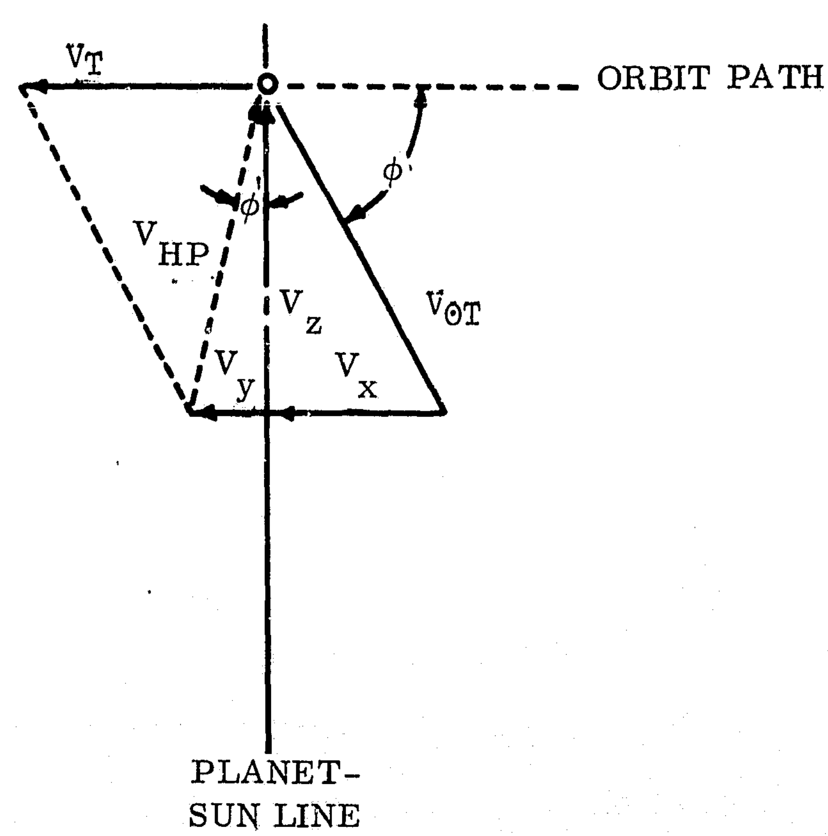
For heliocentric approach angle at the target:

$$\phi = \tan^{-1}\left(\frac{\mu_\odot R_t \epsilon}{R_p^2 V_P^2} \sin \theta\right) . \qquad (12)$$

For heliocentric approach velocity,

$$V_{\odot T} = \left[V_P^2 - 2\left(\frac{\mu_\odot}{R_p} - \frac{\mu_\odot}{R_t}\right)\right]^{1/2} . \qquad (13)$$

The swingby itself must be analyzed in planetocentric coordinates. The velocity of the planet about the sun will be comparable to the velocity of the spacecraft and cannot be ignored. The velocity diagram below, drawn in heliocentric coordinates, defines the relevant terms. V_T is the planet's velocity around the sun. V_x and V_z are components of $V_{\odot T}$. Note that $V_x + V_y = V_T$. V_{HP} is the asymptotic planetocentric approach velocity. The case shown is a retrograde swingby.



The computer routine is as follows:

$$V_x = V_{\theta T} \cos \phi \tag{14}$$

$$V_z = V_{\theta T} \sin \phi \tag{15}$$

$$V_y = |V_T - V_x| \tag{16}$$

$$\phi' = \tan^{-1} (V_y/V_z) \tag{17}$$

$$V_{HP} = (V_y^2 + V_z^2)^{1/2} \tag{18}$$

$$\text{If } V_x < V_T, \text{ retrograde swingby; } \xi = \pi - \phi' \tag{19}$$

$$\text{If } V_x > V_T, \text{ direct swingby; } \xi = \pi + \phi' \tag{20}$$

The departure vector diagram is similar in nature to the approach diagram, and the approach and departure are symmetric around the Earth-sun line in both heliocentric and planetocentric coordinates. Thus, $|\phi'_{APP}| = |\phi'_{DEP}|$ and the required periapsis radius for this amount of gravity turning is:

$$R_{pt} = (\csc \phi' - 1) \frac{\mu_T}{V_{HP}^2} \quad . \tag{21}$$

A minimum allowable value of R_{pt}, R_{pmin}, is established by the extent of the planet's atmosphere. If the above-calculated R_{pt} is less than R_{pmin}, a free-return swingby is not possible and propulsion must be used. There are four parameters of interest:

1. Propulsive ΔV for capture into a very loose orbit.

2. Propulsive ΔV for capture into a circular orbit.

3. Amount of apsidal rotation required for a swingby to Earth return.

4. Propulsive ΔV for this apsidal rotation.

If $R_{pt} < R_{pmin}$, we set $R_{pt} = R_{pmin}$.

Periapsis velocity is found from

$$V_{PT} = \left[2\mu_t / R_{pt} + V_{HP}^2 \right]^{1/2} . \tag{22}$$

Velocity "just captured" at periapsis is

$$V_c = \left(\frac{2\mu_t}{R_{pt}} \right)^{1/2} \tag{23}$$

and velocity for circular capture is

$$V_{cc} = \left(\frac{\mu_t}{R_{pt}} \right)^{1/2} . \tag{24}$$

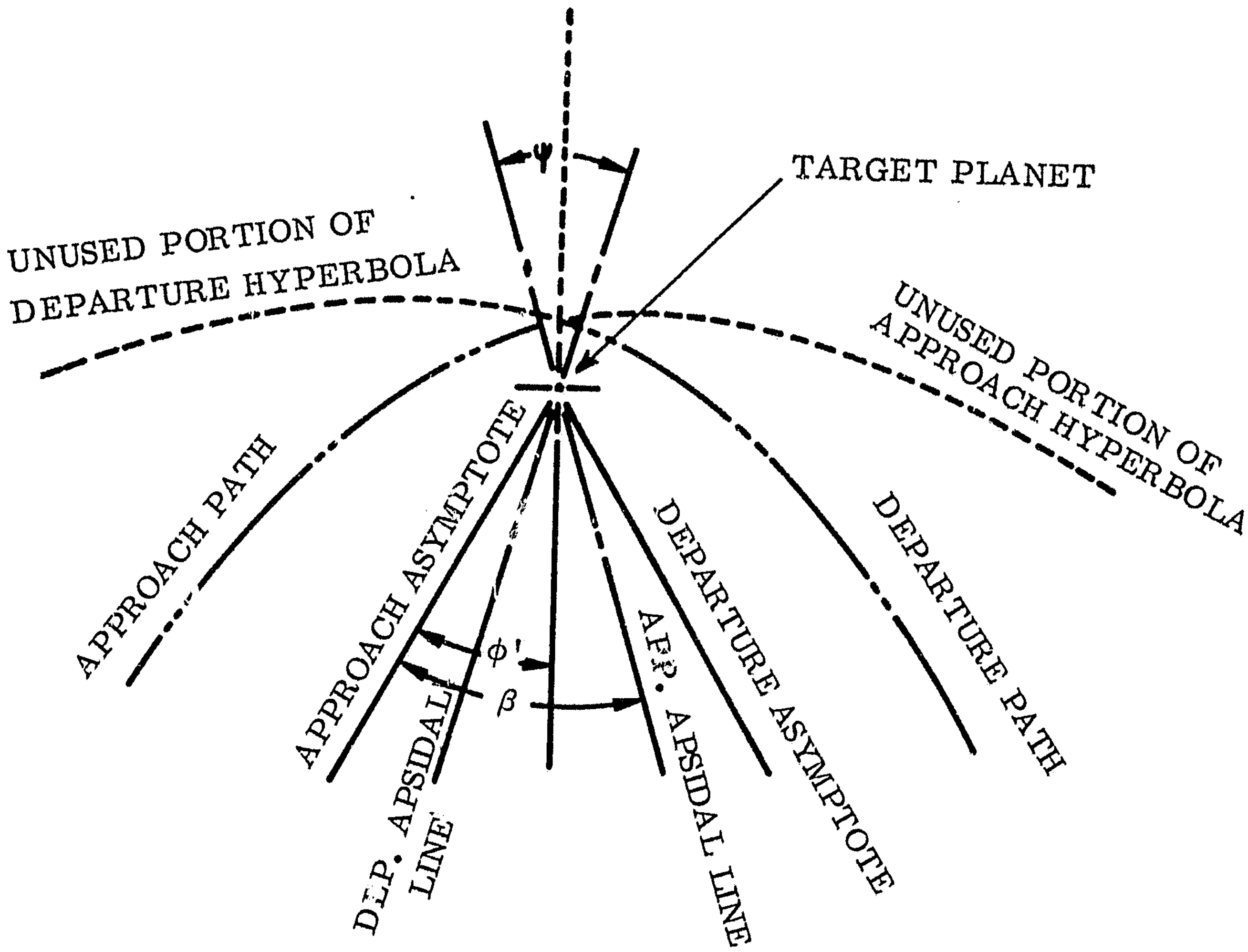
Then the propulsive ΔV's are

$$\Delta V_c = V_{PT} - V_c \tag{25}$$

and

$$\Delta V_{cc} = V_{PT} - V_{cc} \tag{26}$$

Apsidal rotation is sketched below (symmetry is assumed).



β is found from

$$\cos \beta = \frac{\mu_t}{R_{pt} V_{HP}^2 + \mu_t} \tag{27}$$

and

$$\psi_r = 2(\beta - \phi') \quad . \tag{28}$$

The planetocentric path angle, ϕ_{ar}, must be turned downward at the point where the approach and departure hyperbolas intersect. Symmetry determines that $\phi'_{ar} = \phi_{ar}$. Parameters of the approach hyperbola are needed.

$$\epsilon_t = \frac{R_{pt} V_{PT}^2}{\mu_T} - 1 \ , \tag{29}$$

$$R_{ar} = \left[\frac{\mu_t}{R_{pt}^2 V_{PT}^2} \left(1 + \epsilon \cos \frac{\psi_r}{2} \right) \right]^{-1} \ , \tag{30}$$

$$\phi_{ar} = \tan^{-1} \left(\frac{\mu_t R_{ar} \epsilon_t}{R_{pt}^2 V_{PT}^2} \sin \frac{\psi_r}{2} \right) \ , \tag{31}$$

$$V_{ar} = \left[V_{PT}^2 - 2 \left(\frac{\mu_t}{R_{pt}} - \frac{\mu_t}{R_{ar}} \right) \right]^{1/2} \tag{32}$$

$$\Delta V_{ar} = 2 V_{ar} \sin \phi_{ar} \ . \tag{33}$$

If the required amount of apsidal rotation is large, aspidal rotation by capture and later ejection from a circular orbit may require less ΔV than that calculated by equation (33).

All that remains is to compute launch conditions from Earth:

$$V_{HP\oplus} = V_P - \text{EMOS} \ , \tag{34}$$

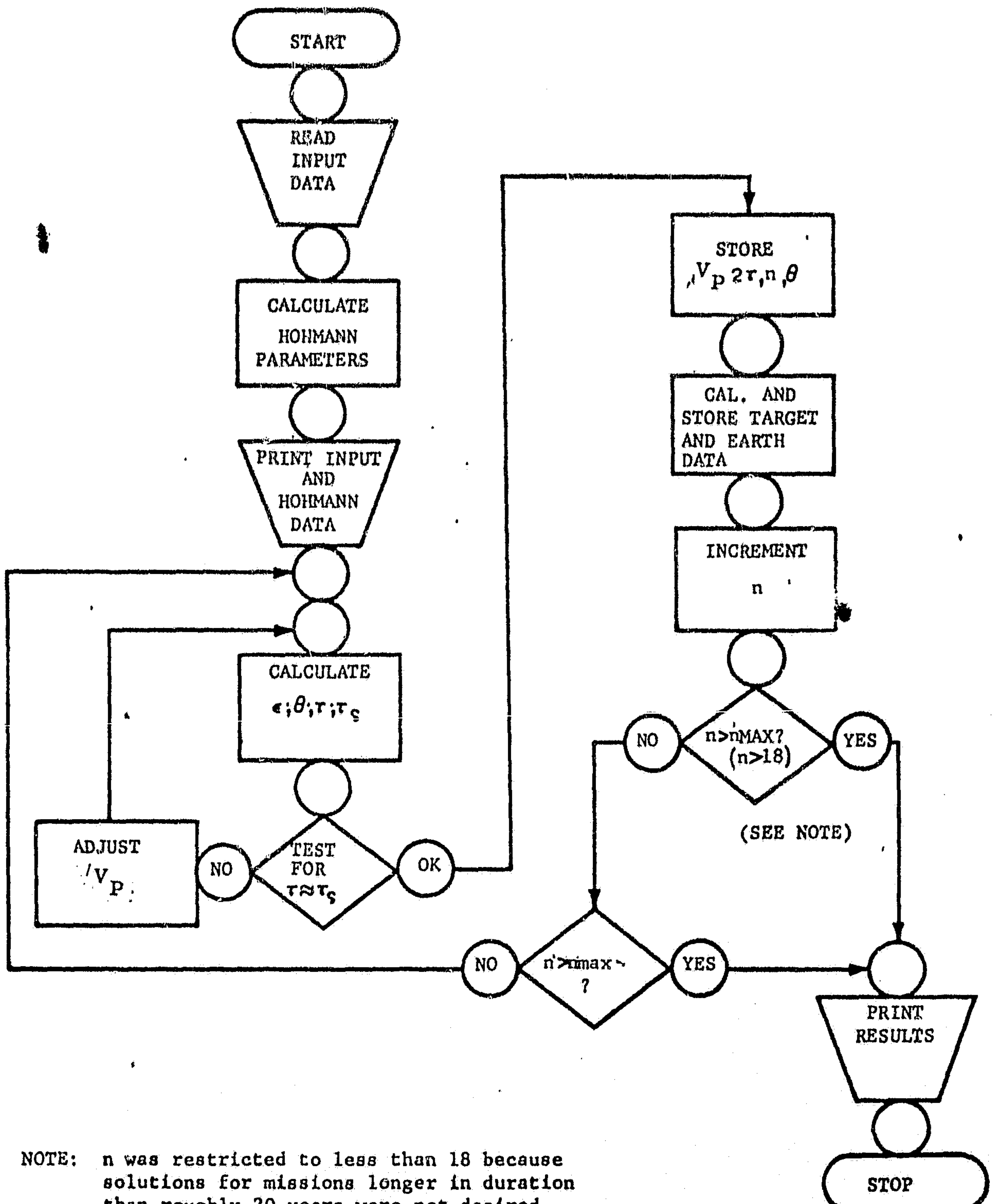
$$V_{PO} = (\mu_\oplus / R_{po})^{1/2} \ , \tag{35}$$

$$V_\oplus = [V_{HP\oplus}^2 + 2\mu_\oplus / R_{po}]^{1/2} \ , \tag{36}$$

$$\Delta V_\oplus = V_\oplus - V_{PO} \ . \tag{37}$$

Note that (by symmetry) $V_{A\oplus} = V_\oplus$.

The overall flow diagram for the computation is shown on the following sheet.



NOTE: n was restricted to less than 18 because solutions for missions longer in duration than roughly 20 years were not desired.

RESULTS

The computer program was exercised for missions to all of the outer planets, beginning with Jupiter. Hand calculations for one of the Jupiter cases ($n = 1$; 600-day mission) were used to check program operation. Missions for $n > 18$ were not analyzed because such long durations were viewed as impractical. Therefore, all possible missions of this symmetric variety were found for Jupiter (4 cases) and Saturn (11 cases), but not for the more distant planets.

Results are summarized in Table I through VI. Free-return missions were found for Jupiter, Saturn, and Uranus. It is presumably true that free-return missions to Nepture and Pluto would have been found if sufficiently long durations had been investigated.

The principal mission of interest for future investigation is the 1000-day flyby of Jupiter. Missions to the other planets are either too long in duration or too demanding in terms of propulsion requirements. The 1000-day Jupiter mission has an impulsive ΔV requirement for launch from Earth orbit of 7.39 km/sec. A reasonable allowance for gravity losses during the escape maneuver would increase this to roughly 8.5 km/sec. This requirement is within the capability of a solid-core (NERVA) nuclear rocket. Entry velocity upon return to Earth is slightly over 15 km/sec, a figure ordinarily viewed as practical for manned Mars missions. This mission could be converted to a loose capture stopover with the expenditure of roughly an additional 4.5 km/sec.

This mission profile is somewhat more difficult than a Mars landing mission in that it is longer (1016 days vs. 450-550 days) and it is exposed to the asteroid belt environment and Jupiter radiation belts environment, both of which are largely unknown. However, the Jupiter flyby incorporates only one major propulsion maneuver.

More energetic propulsion than NERVA-class would allow missions of reasonable duration to the more distant planets. What form this more energetic propulsion might take is not clear at present. Nonetheless, assuming it to be of some high-thrust form such that the impulsive-maneuver calculations employed herein are relevant, mission ΔV requirements as a function of target planet distance can be plotted on the basis of an assumed mission duration of roughly five years (Fig. 3). The rapid escalation of propulsion requirement with greater distance is striking. For Uranus, Neptune, and Pluto, less propulsion is required to capture into a circular orbit and re-eject than to perform an apsidal rotation maneuver.

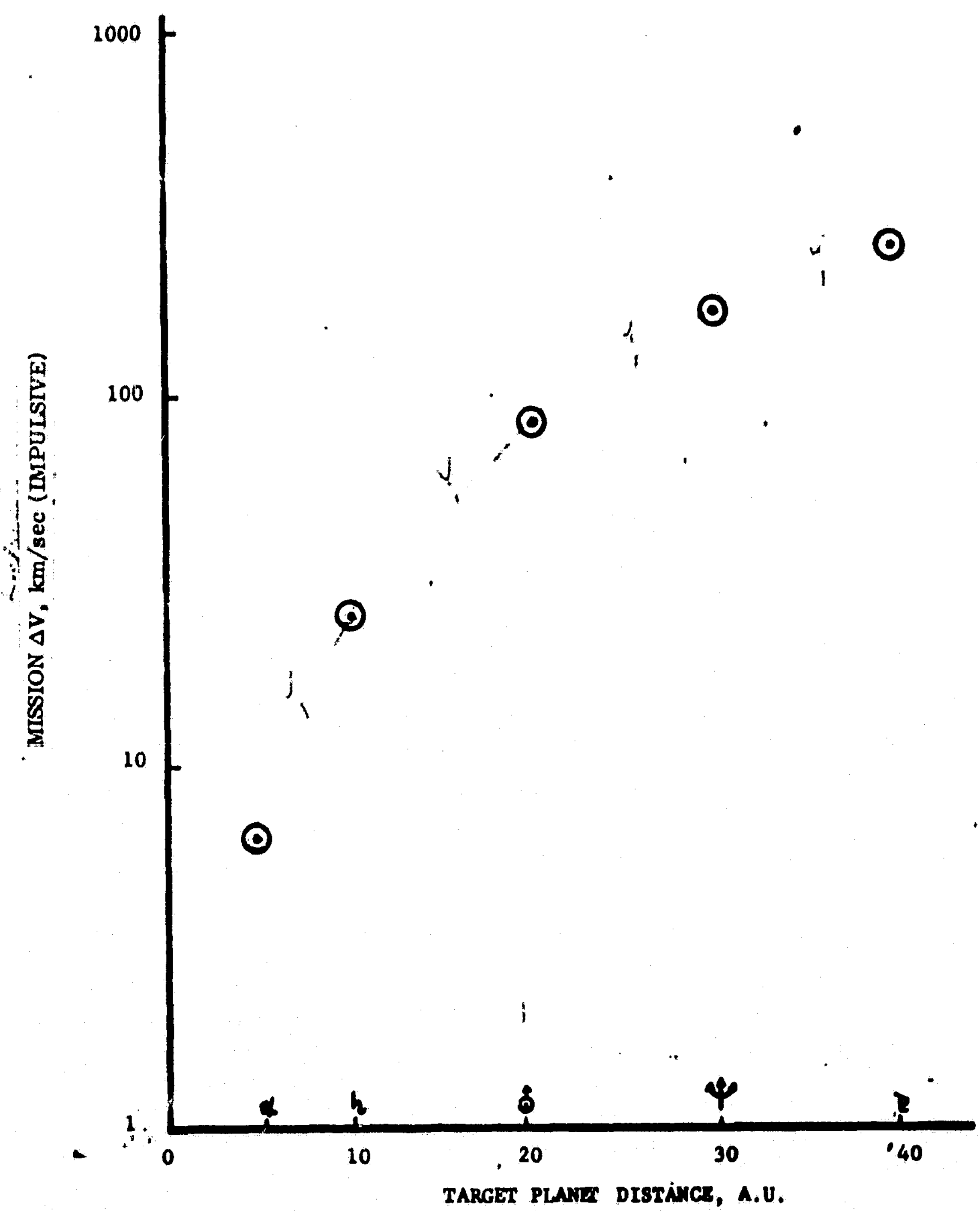


FIGURE 3. PROPULSION REQUIREMENT AS A FUNCTION OF DISTANCE FOR FIVE-YEAR ROUNDTRIP MISSIONS

RECOMMENDED ADDITIONAL STUDY

The 1000-day Jupiter flyby merits further study. The first step should be more precise flight mechanics analysis, using actual planetary orbits, to check effects of launch window requirements and to insure that, under more precise analysis, the periapsis altitude required at Jupiter will remain safely above the atmosphere.

The reason that rapid missions to more distant planets are not free-returns is, at least partly, that the required gravity path deflection at the target approximates 180°. A multiplanet mission could possibly be contrived to reduce propulsion requirements as compared to a single-planet flyby. Figure 4 shows a postulated Jupiter-Saturn flyby. This mission was not analyzed, but the required turning angles appear possible with a low-periapsis (70 000 km) flyby of Saturn, flying between the rings and the planet surface. A two-dimensional analysis is recommended as a first step in checking the feasibility of this mission. It appears possible that the Jupiter swingby would reduce the propulsion requirements for a five-year Saturn round trip mission from about 26 km/sec to about 12 km/sec.

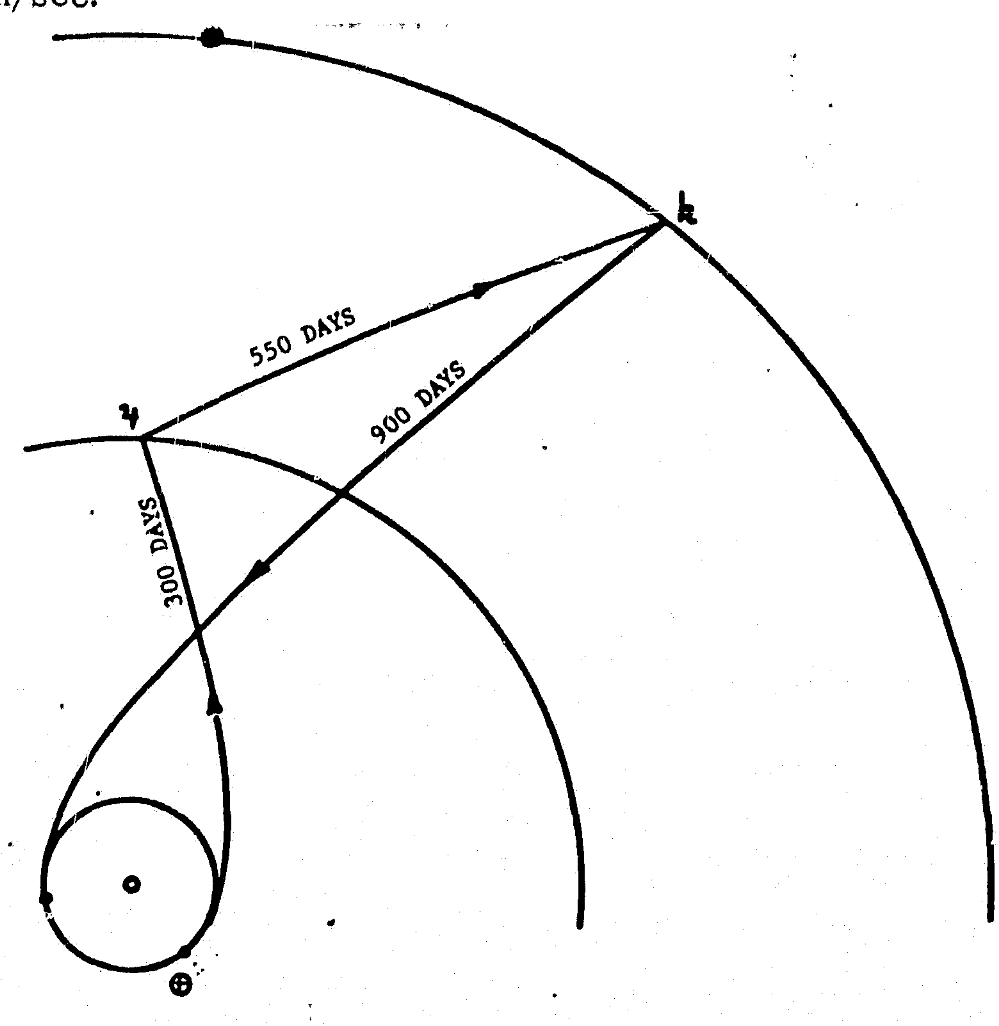


FIGURE 4. HYPOTHETICAL FIVE-YEAR JUPITER-SATURN FLYBY

TABLE I. INPUT DATA

Solar Gravitational Constant	=	1.32511E 11	km^3/sec^2
Earth Gravitational Constant	=	3.98603E 05	km^3/sec^2
Radius to Earth Orbit	=	1.49600E 08	km
Earth Mean Orbital Speed	=	29.80	km/sec
Radius to Earth Parking Orbit	=	6663.00	km
Altitude of Earth Parking Orbit	=	285.00	km

	Jupiter	Saturn	Uranus	Neptune	Pluto
Gravitational Const, km^3/sec^2	1.264 E8	3.786 E7	5.812 E6	6.796 E6	3.786 E5
Radius to Orbit, km	7.79 E8	1.428 E9	2.865 E9	4.49 E9	5.90 E9
Allowed Minimum Periapsis Radius for Flyby, km	80 000	70 000	30 000	30 000	40 000
Hohmann Transfer:					
Ellipse Eccentricity	0.6778	0.8103	0.9007	0.9355	0.9505
One-Way Duration (Days)	999	2213	5845	11 161	16 617
Perihelion Velocity, km/sec	38.55	40.04	41.03	41.41	41.57

NOTE: In following Tables, velocities are in km/sec, duration in days, angles in radians

TABLE II. RESULTS FOR SYMMETRIC FLYBY MISSIONS - JUPITER

Number of Earth Revolutions	Mission Duration	Heliocentric Velocity at Perigee	Hyperbolic Excess Velocity at Earth	Impulsive ΔV	Aerodynamic Entry Speed at Earth	Eccentricity of Transfer Path	One-Way Transfer Angle
1	591.4965	46.5859	16.7859	12.3007	20.0353	1.4501	1.9446
2	1016.0310	40.2405	10.4405	7.3866	15.1212	.6281	2.4713
3	1423.3218	38.8805	9.0805	6.4817	14.2162	.7066	2.8283
4	1819.0016	38.5715	8.7715	6.2863	14.0209	.6796	3.0615

Hyperbolic Excess Velocity at Target	Heliocentric Approach Angle at Target	Planetocentric Approach Angle at Target	Periapsis Altitude at Target	ΔV for Capture at Target	ΔV for Circular Capture at Target	Amount of Apsidal Rotation for Flyby	ΔV for Apsidal Rotation for Flyby
2.5999E 01	1.2354E 00	2.9819E 00	2.4091E 03	5.7213E 00	2.2186E 01	1.2704E 00	4.3225E 01
1.4506E 01	1.1435E 00	2.6064E 00	9.7658E 04	2.0274E 00	1.6929E 01	0	0
1.2119E 01	1.1086E 00	2.2944E 00	4.3926E 05	2.8874E 00	9.9139E 00	0	0
1.1758E 01	1.0993E 00	2.1763E 00	6.9206E 05	3.3274E 00	8.9253E 00	0	0

TABLE III. RESULTS FOR SYMMETRIC FLYBY MISSIONS - SATURN

Number of Earth Revolutions	Mission Duration	Heliocentric Velocity at Perigee	Hyperbolic Excess Velocity at Earth	Impulsive ΔV	Aerodynamic Entry Speed at Earth	Eccentricity of Transfer Path	One-Way Transfer Angle
1	559.1649	69.6772	39.8772	33.615[illegible]	41.3502	4.4810	1.6660
2	1005.5356	49.9443	20.1443	15.1879	22.9225	1.8161	1.9728
3	1352.0258	44.9788	15.1788	10.9748	18.7094	1.2840	2.2049
4	1743.7828	42.5020	12.7020	9.0282	16.7627	1.0394	2.4268
5	2129.2041	41.3228	11.5228	8.1532	15.8878	.9278	2.6064
6	2510.6799	40.7102	10.9102	7.7147	15.4493	.8711	2.7466
7	2890.7859	40.3773	10.5773	7.4814	15.2160	.8406	2.8588
8	3264.3243	40.1976	10.3976	7.3570	15.0916	.8242	2.9484
9	3638.4173	40.1022	10.3022	7.2915	15.0260	.8156	3.0224
10	4011.6850	40.0577	10.2577	7.2610	14.9956	.8116	3.0841
11	4383.3051	40.0444	10.2444	7.2519	14.9865	.8104	3.1362

Hyperbolic Excess Velocity at Target	Heliocentric Approach Angle at Target	Planetocentric Approach Angle at Target	Periapsis Altitude at Target	ΔV for Capture at Target	ΔV for Circular Capture at Target	Amount of Apsidal Rotation for Flyby	ΔV for Apsidal Rotation for Flyby
5.6756E 01	1.4428E 00	3.1002E 00	1.0092E 01	3.2707E 01	4.2340E 01	2.7702E 00	1.1273E 02
3.0132E 01	1.4090E 00	2.9818E 00	5.3839E 02	1.1716E 01	2.1349E 01	2.0568E 00	5.5636E 01
2.1354E 01	1.3857E 00	2.8664E 00	3.2463E 03	6.3245E 00	1.5958E 01	1.4442E 00	3.3606E 01
1.5989E 01	1.3672E 00	2.7126E 00	1.4740E 04	3.6805E 00	1.3314E 01	7.9117E-01	1.6786E 01
1.3011E 01	1.3555E 00	2.5465E 00	4.6435E 04	2.4800E 00	1.2113E 01	2.1970E-01	4.4023E 00
1.1338E 01	1.3485E 00	2.3845E 00	1.1071E 05	2.3518E 00	1.0012E 01	0	0
1.0472E 01	1.3443E 00	2.2431E 00	2.1107E 05	2.6782E 00	8.2258E 00	0	0
9.9539E 00	1.3420E 00	2.1353E 00	3.3214E 05	2.9858E 00	7.4082E 00	0	0
9.7263E 00	1.3407E 00	2.0624E 00	4.4756E 05	3.2344E 00	7.0440E 00	0	0
9.6293E 00	1.3401E 00	2.0229E 00	5.2633E 05	3.3871E 00	6.9001E 00	0	0
9.6019E 00	1.3399E 00	2.0102E 00	5.5461E 05	3.4391E 00	6.8614E 00	0	0

TABLE IV. RESULTS FOR SYMMETRIC FLYBY MISSIONS - URANUS
(Sheet 1 of 2)

Number of Earth Revolutions	Mission Duration	Heliocentric Velocity at Perigee	Hyperbolic Excess Velocity at Earth	Impulsive ΔV	Aerodynamic Entry Speed at Earth	Eccentricity of Transfer Path	One-Way Transfer Angle
1	548.3367	126.9330	97.1330	90.0124	97.7469	17.1898	1.5737
2	924.1305	81.6289	51.8289	45.2360	52.9705	6.5226	1.6640
3	1302.6084	64.1645	34.3645	28.3288	36.0633	3.6480	1.7799
4	1682.9347	55.5566	25.7566	20.2485	27.9831	2.4846	1.9063
5	2060.1776	50.7710	20.9710	15.9177	23.6523	1.9101	2.0308
6	2440.6964	47.8261	18.0261	13.3507	21.0852	1.5823	2.1493
7	2819.0230	45.9320	16.1320	11.7561	19.4907	1.3818	2.2571
8	3194.7017	44.6565	14.8565	10.7144	18.4489	1.2514	2.3535
9	3570.6666	43.7597	13.9597	10.0002	17.7347	1.1619	2.4396
10	3945.0649	43.1146	13.3146	9.4970	17.2315	1.0986	2.5158
11	4318.5700	42.6390	12.8390	9.1321	16.8667	1.0525	2.5835
12	4692.0257	42.2810	12.4810	8.8613	16.5958	1.0182	2.6437
13	5061.6607	42.0100	12.2100	8.6584	16.3930	.9924	2.6970
14	5433.0486	41.7991	11.9991	8.5020	16.2365	.9725	2.7450
15	5803.2540	41.6347	11.8347	8.3809	16.1154	.9570	2.7881
16	6172.9738	41.5053	11.7053	8.2861	16.0206	.9449	2.8269
17	6542.2716	41.4027	11.6027	8.2113	15.9459	.9353	2.8620
18	6911.2015	41.3211	11.5211	8.1520	15.8866	.9276	2.8940

TABLE IV. RESULTS FOR SYMMETRIC FLYBY MISSIONS - URANUS
(Sheet 2 of 2)

Hyperbolic Excess Velocity at Target	Heliocentric Approach Angle at Target	Planetocentric Approach Angle at Target	Periapsis Altitude at Target	ΔV for Capture at Target	ΔV for Circular Capture at Target	Amount of Apsidal Rotation for Flyby	ΔV for Apsidal Rotation for Flyby
1.1995E 02	1.5156E 00	3.1402E 00	4.1530E-04	1.0187E 02	1.0764E 02	3.1122E 00	2.3990E 02
7.0518E 01	1.5106E 00	3.1053E 00	7.6855E-01	5.3529E 01	5.9295E 01	2.9941E 00	1.4075E 02
4.9393E 01	1.5044E 00	3.0702E 00	6.0801E 00	3.3487E 01	3.9252E 01	2.8516E 00	9.8016E 01
3.7636E 01	1.4977E 00	3.0335E 00	2.4083E 01	2.2789E 01	2.8554E 01	2.6842E 00	7.3849E 01
3.0209E 01	1.4914E 00	2.9947E 00	6.9372E 01	1.6372E 01	2.2137E 01	2.4957E 00	5.8193E 01
2.5021E 01	1.4858E 00	2.9524E 00	1.6867E 02	1.2152E 01	1.7917E 01	2.2860E 00	4.6860E 01
2.1252E 01	1.4810E 00	2.9070E 00	3.6235E 02	9.2833E 00	1.5049E 01	2.0627E 00	3.8231E 01
1.8407E 01	1.4771E 00	2.8586E 00	7.1044E 02	7.2657E 00	1.3031E 01	1.8310E 00	3.1336E 01
1.6184E 01	1.4739E 00	2.8070E 00	1.3029E 03	5.7989E 00	1.1564E 01	1.5941E 00	2.5580E 01
1.4421E 01	1.4714E 00	2.7526E 00	2.2560E 03	4.7175E 00	1.0483E 01	1.3572E 00	2.0667E 01
1.3002E 01	1.4693E 00	2.6958E 00	3.7234E 03	3.9062E 00	9.6716E 00	1.1234E 00	1.6381E 01
1.1844E 01	1.4677E 00	2.6369E 00	5.8999E 03	3.2888E 00	9.0542E 00	8.9482E-01	1.2581E 01
1.0905E 01	1.4663E 00	2.5771E 00	8.9770E 03	2.8186E 00	8.5840E 00	6.7588E-01	9.2127E 00
1.0126E 01	1.4653E 00	2.5159E 00	1.3247E 04	2.4520E 00	8.2174E 00	4.6478E-01	6.1649E 00
9.4870E 00	1.4644E 00	2.4546E 00	1.8947E 04	2.1669E 00	7.9323E 00	2.6444E-01	3.4238E 00
8.9606E 00	1.4637E 00	2.3937E 00	2.6349E 04	1.9436E 00	7.7089E 00	7.5376E-02	9.5479E-01
8.5283E 00	1.4632E 00	2.3339E 00	3.5701E 04	1.9139E 00	7.1989E 00	0	0
8.1745E 00	1.4627E 00	2.2759E 00	4.7219E 04	2.0018E 00	6.5972E 00	0	0

TABLE V. RESULTS FOR SYMMETRIC FLYBY MISSIONS - NEPTUNE
(Sheet 1 of 2)

Number of Earth Revolutions	Mission Duration	Heliocentric Velocity at Perigee	Hyperbolic Excess Velocity at Earth	Impulsive ΔV	Aerodynamic Entry Speed at Earth	Eccentricity of Transfer Path	One-Way Transfer Angle
1	546.8847	194.0780	164.2780	156.9072	164.6417	41.5238	1.5608
2	915.6543	120.2211	90.4211	83.3457	91.0803	15.3170	1.6006
3	1289.9690	89.8611	60.0611	53.3145	61.0490	8.1164	1.6567
4	1661.5431	74.1697	44.3697	37.9636	45.6961	5.2106	1.7236
5	2033.4505	64.8437	35.0437	28.9766	36.7112	3.7470	1.7974
6	2408.7361	58.8026	29.0026	23.2622	30.9967	2.9037	1.8751
7	2783.4771	54.7151	24.9151	19.4759	27.2105	2.3798	1.9529
8	3157.7517	51.8300	22.0300	16.8616	24.5961	2.0328	2.0289
9	3531.5475	49.7258	19.9258	14.9961	22.7307	1.7915	2.1016
10	3903.5434	48.1548	18.3548	13.6324	21.3669	1.6179	2.1702
11	4277.3340	46.9446	17.1446	12.6022	20.3368	1.4880	2.2349
12	4649.7919	46.0034	16.2034	11.8153	19.5499	1.3892	2.2950
13	5020.8271	45.2595	15.4595	11.2033	18.9379	1.3126	2.3506
14	5392.8255	44.6596	14.6596	10.7168	18.4514	1.2517	2.4024
15	5763.6772	44.1729	14.3729	10.3272	18.0617	1.2029	2.4501
16	6134.1612	43.7730	13.9730	10.0106	17.7452	1.1632	2.4944
17	6504.3358	43.4412	13.6412	9.7506	17.4851	1.1305	2.5353
18	6874.2776	43.1637	13.3637	9.5350	17.2695	1.1034	2.5732

TABLE V. RESULTS FOR SYMMETRIC FLYBY MISSIONS - NEPTUNE
(Sheet 2 of 2)

Hyperbolic Excess Velocity at Target	Heliocentric Approach Angle at Target	Planetocentric Approach Angle at Target	Periapsis Altitude at Target	ΔV for Capture at Target	ΔV for Circular Capture at Target	Amount of Apsidal Rotation for Flyby	ΔV for Apsidal Rotation for Flyby
1.8951E 02	1.5367E 00	3.1470E 00	2.7756E-03	1.6941E 02	1.7565E 02	3.1182E 00	3.7900E 02
1.1281E 02	1.5353E 00	3.1289E 00	4.3267E-02	9.3517E 01	9.9752E 01	3.0812E 00	2.2556E 02
7.9747E 01	1.5334E 00	3.1108E 00	5.0748E-01	6.1254E 01	6.7488E 01	3.0112E 00	1.5925E 02
6.1576E 01	1.5311E 00	3.0929E 00	2.1273E 00	4.3866E 01	5.0101E 01	2.9314E 00	1.2267E 02
4.9989E 01	1.5286E 00	3.0748E 00	6.0709E 00	3.3047E 01	3.9281E 01	2.8417E 00	9.9199E 01
4.1887E 01	1.5260E 00	3.0562E 00	1.4150E 01	2.5699E 01	3.1934E 01	2.7417E 00	8.2646E 01
3.5950E 01	1.5235E 00	3.0371E 00	2.8813E 01	2.0494E 01	2.6728E 01	2.6333E 00	7.0378E 01
3.1410E 01	1.5211E 00	3.0174E 00	5.3460E 01	1.6657E 01	2.2892E 01	2.5175E 00	6.0857E 01
2.7824E 01	1.5189E 00	2.9970E 00	9.2632E 01	1.3747E 01	1.9981E 01	2.3956E 00	5.3203E 01
2.4931E 01	1.5169E 00	2.9758E 00	1.5200E 02	1.1496E 01	1.7730E 01	2.2692E 00	4.6897E 01
2.2527E 01	1.5151E 00	2.9537E 00	2.3983E 02	9.7071E 00	1.5941E 01	2.1384E 00	4.1531E 01
2.0516E 01	1.5136E 00	2.9309E 00	3.6510E 02	8.2774E 00	1.4512E 01	2.0053E 00	3.6919E 01
1.8810E 01	1.5122E 00	2.9074E 00	5.3928E 02	7.1203E 00	1.3355E 01	1.8711E 00	3.2891E 01
1.7339E 01	1.5109E 00	2.8829E 00	7.7802E 02	6.1681E 00	1.2402E 01	1.7359E 00	2.9302E 01
1.6065E 01	1.5099E 00	2.8578E 00	1.0975E 03	5.3821E 00	1.1616E 01	1.6011E 00	2.6087E 01
1.4952E 01	1.5089E 00	2.8318E 00	1.5190E 03	4.7266E 00	1.0961E 01	1.4672E 00	2.3171E 01
1.3972E 01	1.5081E 00	2.8052E 00	2.0671E 03	4.1759E 00	1.0410E 01	1.3347E 00	2.0504E 01
1.3104E 01	1.5074E 00	2.7779E 00	2.7704E 03	3.7105E 00	9.9448E 00	1.2040E 00	1.8046E 01

TABLE VI. RESULTS FOR SYMMETRIC FLYBY MISSIONS - PLUTO
(Sheet 1 of 2)

Number of Earth Revolutions	Mission Duration	Heliocentric Velocity at Perigee	Hyperbolic Excess Velocity at Earth	Impulsive ΔV	Aerodynamic Entry Speed at Earth	Eccentricity of Transfer Path	One-Way Transfer Angle
1	548.0403	252.3537	222.5537	215.0878	222.8223	70.8950	1.5592
2	914.[illegible]666	154.6779	124.8779	117.6215	125.3560	26.0107	1.5829
3	1285.[illegible]159	113.6200	83.8200	76.7961	84.5307	13.5743	1.6173
4	1654.1021	91.8418	62.0418	55.2641	62.9987	8.5227	1.6599
5	2023.4429	78.5883	48.7883	42.2649	49.9994	5.9726	1.7091
6	2395.6114	69.8028	40.0028	33.7367	4[illegible].4713	4.5008	1.7632
7	2768.0303	63.7139	33.9139	27.8997	35.6343	3.5830	1.8200
8	3140.1986	59.3195	29.5195	23.7464	31.4809	2.9726	1.8781
9	3512.2874	56.0485	26.2485	20.7018	28.4364	2.5466	1.9363
10	3884.2305	53.5528	23.7528	18.4158	26.1504	2.2378	1.9935
11	4254.3356	51.6171	21.8171	16.6710	24.4056	2.0079	2.0488
12	4626.6981	50.0729	20.2729	15.3010	23.0355	1.8306	2.1026
13	4998.4719	48.8329	19.0329	14.2176	21.9522	1.6922	2.1539
14	5369.9653	47.8233	18.0233	13.3483	21.0828	1.5820	2.2027
15	5740.0465	46.9941	17.1941	12.6439	20.3785	1.4933	2.2489
16	6109.6188	46.3039	16.5039	12.0651	19.7996	1.4206	2.2926
17	6478.8708	45.7248	15.9248	11.5850	19.3196	1.3604	2.3338
18	6850.4056	45.2310	15.4310	11.1801	18.9146	1.3097	2.3729

TABLE VI. RESULTS FOR SYMMETRIC FLYBY MISSIONS - PLUTO
(Sheet 2 of 2)

Hyperbolic Excess Velocity at Target	Heliocentric Approach Angle at Target	Planetocentric Approach Angle at Target	Periapsis Altitude at Target	ΔV for Capture at Target	ΔV for Circular Capture at Target	Amount of Apsidal Rotation for Flyby	ΔV for Apsidal Rotation for Flyby
2.4883E 02	1.5451E 00	3.1483E 00	1.3585E-04	2.4452E 02	2.4579E 02	3.1280E 00	4.9765E 02
1.4894E 02	1.5445E 00	3.1361E 00	2.5740E-04	1.4466E 02	1.4593E 02	3.1298E 00	2.9786E 02
1.0573E 02	1.5436E 00	3.1240E 00	5.2537E-03	1.0146E 02	1.0274E 02	3.1047E 00	2.1142E 02
8.1907E 01	1.5425E 00	3.1120E 00	2.4655E-02	7.7672E 01	7.6946E 01	3.0797E 00	1.6374E 02
6.6733E 01	1.5412E 00	3.1001E 00	7.3134E-02	6.2524E 01	6.3798E 01	3.0544E 00	1.3354E 02
5.6141E 01	1.5398E 00	3.0881E 00	1.7217E-01	5.1958E 01	5.3233E 01	3.0286E 00	1.1210E 02
4.8378E 01	1.5384E 00	3.0759E 00	3.4932E-01	4.4223E 01	4.5497E 01	3.0022E 00	9.6523E 01
4.2439E 01	1.5369E 00	3.0636E 00	6.4082E-01	3.8310E 01	3.9585E 01	2.9752E 00	8.4565E 01
3.7745E 01	1.5355E 00	3.0511E 00	1.0927E 00	3.3644E 01	3.4919E 01	2.9473E 00	7.5136E 01
3.3941E 01	1.5341E 00	3.0382E 00	1.7628E 00	2.9868E 01	3.1142E 01	2.9186E 00	6.7405E 01
3.0808E 01	1.5328E 00	3.0252E 00	2.7170E 00	2.6763E 01	2.8038E 01	2.8891E 00	6.1129E 01
2.8156E 01	1.5316E 00	3.0118E 00	4.0535E 00	2.4139E 01	2.5414E 01	2.8583E 00	5.5752E 01
2.5898E 01	1.5305E 00	2.9980E 00	5.8699E 00	2.1910E 01	2.3184E 01	2.8266E 00	5.1160E 01
2.3951E 01	1.5294E 00	2.9839E 00	8.2913E 00	1.9992E 01	2.1266E 01	2.7938E 00	4.7185E 01
2.2260E 01	1.5285E 00	2.9695E 00	1.1454E 01	1.8330E 01	1.9604E 01	2.7600E 00	4.3719E 01
2.0773E 01	1.5276E 00	2.9548E 00	1.5533E 01	1.6873E 01	1.8148E 01	2.7251E 00	4.0659E 01
1.9459E 01	1.5268E 00	2.9397E 00	2.0721E 01	1.5589E 01	1.6863E 01	2.6891E 00	3.7938E 01
1.8279E 01	1.5261E 00	2.9242E 00	2.7302E 01	1.4439E 01	1.5713E 01	2.6518E 00	3.5481E 01

*TOTAL 3200 CPU TIME 0.036 HOURS, TOTAL OPERATING EXPENSE $ 4.86 **

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